Delft University of Technology, EEMCS faculty
Examination Mathematics 2, AESB1210-15
Tuesday, January 30th, 2018, 9.00-12.00

- It's not allowed to use a calculator or a mathematical table.
- Each answer should be clearly motivated.
- Simplify your answer as much as possible.
- Your grade is obtained by rounding (score+6)/6 to the nearest half.
- Points:

Ex. 1	6	Ex. 2	6	Ex. 3	5	Ex. 4	4	Ex. 5a	3	Ex. 6	3,5
								Ex. 5b	2,5		
Ex. 7a	2,5	Ex. 8	3	Ex. 9a	3	Ex. 10	3	Ex. 11	3,5		
Ex. 7b	2,5			Ex. 9b	2						
Ex. 7c	2,5			Ex. 9c	2						

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1. Find the general solution, in explicit form, of $xy' + \dfrac{x}{x+1}y = 5x^3$ if $x > 0$.

2. Find the solution, in explicit form, of the initial-value problem $\begin{cases} y' = \dfrac{x}{y + x^2 y} \\ y(0) = -2 \end{cases}$.

3. Find all solutions, in the form $a + bi$ with $a, b \in \mathbb{R}$, of $z^3 + i = 0$.

4. Find the general solution, in real form, of $y'' + 2ay' + (a^2 + 1)y = 0$, where $a \in \mathbb{R}$.

5. The linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ is defined by the formula

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 + x_3 \\ x_2 + 3x_3 \\ x_2 - x_3 \\ 3x_1 + 6x_2 + \beta x_3 \end{bmatrix}, \text{ where } \beta \in \mathbb{R}$$

a. For which value(s) of β is vector $\underline{b} = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 5 \end{bmatrix}$ in the range of T?

b. For which value(s) of β is transformation T one-to-one ?

P.T.O.

6. Find a basis for subspace $H = \left\{ \begin{bmatrix} r+3s+4t \\ -s-t \\ -r-t \\ 2r+4s+6t \end{bmatrix} \middle| \, r,s,t \in \mathbb{R} \right\}$ of $\mathbb{R}^4$.

7. Mark each statement True or False, justify your answers.

   a. **Statement 1**: If $\{\underline{b}_1, \underline{b}_2, \underline{b}_3\}$ is linearly independent, $\underline{b}_4 = \underline{b}_1 + \underline{b}_2$, $\underline{b}_5 = \underline{b}_1 + \underline{b}_2 + \underline{b}_3$ and matrix $B = [\underline{b}_1\ \underline{b}_2\ \underline{b}_3\ \underline{b}_4\ \underline{b}_5]$, so matrix B has 5 columns, then $dim(NUL(B)) = 3$.

   b. **Statement 2**: If A is an $n \times n$ matrix and matrix B is the inverse of A^2 then AB is the inverse of A.

   c. **Statement 3**: If $\underline{a}, \underline{b}, \underline{c} \in \mathbb{R}^n$, $H = Span\{\underline{a}, \underline{b}, \underline{c}\}$ and $\underline{x} \in \mathbb{R}^n$, such that $\underline{x} \perp \underline{a}$, $\underline{x} \perp \underline{b}$ and $\underline{x} \perp \underline{c}$ then $\underline{x} \in H^{\perp}$ (the orthogonal complement of H in $\mathbb{R}^n$).

8. Suppose C is a 3×6 – matrix and $\{\underline{u}_1, \underline{u}_2, \underline{u}_3, \underline{u}_4\}$ is a basis for $NUL(C)$. Find p and q such that the following statement is true:
   The column space of C is a p – dimensional subspace of $\mathbb{R}^q$.
   (So determine p and q, justify your answer)

9. Let $\underline{b}_1 = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \underline{b}_2 = \begin{bmatrix} 3 \\ 2 \\ 2 \\ -1 \end{bmatrix}, \underline{y} = \begin{bmatrix} 7 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ and $W = Span\{\underline{b}_1, \underline{b}_2\}$.

   a. Find the projection of $\underline{y}$ onto W.

   b. Find a matrix F such that $W^{\perp} = NUL(F)$.

   c. Find a basis for $W^{\perp}$.

10. Let $U = [\underline{a}\ \underline{b}\ \underline{c}]$ be an $m \times 3$ matrix (so U has 3 columns, $\underline{a}$, $\underline{b}$ and $\underline{c}$) such that
   $U^T U = \begin{bmatrix} 16 & -2 & 9 \\ -2 & 25 & -10 \\ 9 & -10 & \sqrt{3} \end{bmatrix}$. Find α if $\alpha\underline{b}$ is the orthogonal projection from $\underline{c}$ onto $\underline{b}$.

11. A certain experiment generates the data $(0,2)$, $(1,1)$, $(2,0)$ and $(3,1)$ in the xy – plane.
   To produce a least-squares fit we consider functions of the form $y = \alpha + \beta x + \gamma x^2$, where $\alpha, \beta, \gamma \in \mathbb{R}$.
   Construct the system of normal equations for finding the parameters α, β and γ.
   (You don't need to solve this system of normal equations!)